University of Zurich

Socioeconomic Institute
Sozialökonomisches Institut

Working Paper No. 0912

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Publisher Sozialökonomisches Institut
Bibliothek (Working Paper)
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Simulating WTP Values from Random-Coefficient Models

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This version: July 28, 2009

Abstract

Discrete Choice Experiments (DCEs) designed to estimate willingness-to-pay (WTP) values are very popular in health economics. With increased computation power and advanced simulation techniques, random-coefficient models have gained an increasing importance in applied work as they allow for taste heterogeneity. This paper discusses the parametrical derivation of WTP values from estimated random-coefficient models and shows how these values can be simulated in cases where they do not have a known distribution.

JEL-Classification: C15, C25

Keywords: willingness-to-pay, discrete choice, simulation, random-coefficient models

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Introduction

Discrete choice experiments (DCEs) have gained increasing importance in applied health economics. They allow to calculate willingness-to-pay (WTP) values for goods which are not (yet) available on the market or for public goods that have no market price. Recently, random-coefficient models have become popular in DCEs because they allow for taste heterogeneity among the respondents, e.g. Revelt and Train [1999], Hole [2008]. Increased computation power and advanced simulation techniques have fostered this development. Hensher and Greene [2003] summarize the state of practice of mixed logit models.

This paper provides an overview of the methods available for calculating WTP values from random-coefficient estimates. In the next section, we discuss the two most commonly used so-called

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mixing distributions. The subsequent section deals with the calculation of the WTP distribution resulting from random-coefficient models. Frequently, the mixing distributions employed do not permit to derive the WTP distribution parametrically. However, simulation provides a solution in these cases, as shown in two illustrative examples.

Mixing Distributions

Estimation of random-coefficient model require the choice of a density defined over the possible coefficient values, the so-called mixing distribution. The two most commonly used are the normal (N) and the lognormal (LN) distributions (see Revelt and Train [1999], Hole [2008]). The univariate N has the form

$$f_N(\beta|\mu,\sigma) = \frac{1}{\sqrt{(2\pi)}\sigma} exp(-\frac{(\beta-\mu)^2}{2\sigma^2}), \tag{1}$$

where μ and σ denote the mean and the standard deviation, respectively. Since, $\beta \sim N(\mu, \sigma^2)$ causes the corresponding attribute to have positive or negative marginal utility, this specification results in a conflict with maintained economic hypotheses. For instance, an attribute may constitute a normal good for some individuals and an inferior one for others. To avoid this conflict, many researchers prefer the LN alternative,

$$f_{LN}(\beta|\mu,\sigma) = \frac{1}{\beta\sqrt{(2\pi)}\sigma} exp(-\frac{(ln(\beta)-\mu)^2}{2\sigma^2}). \tag{2}$$

It is of advantage to keep in mind that the N and LN distributions are related in the following way. If $b \sim N(\mu, \sigma^2)$, then

$$\beta \sim exp(b) \tag{3}$$

is LN distributed. For more details, see Poirier [1995].

Derivation of WTP Values

As long as the attributes enter the utility function in linear form, WTP for attribute k is a constant given by

$$WTP_k = -\frac{\beta_k}{\beta_p}, \tag{4}$$

where β_k is the coefficient of the attribute of interest and $\beta_p < 0$ belongs to the price attribute. With fixed-coefficient models, the β's do not vary across individuals and the calculation of the WTP are straight forward applying Eq. (4).

In the case of random-coefficient models, Eq. (4) calls for dividing the two estimated mixing distributions by each other. In general, this division does not result in a well-specified distribution.

It has only a known form (LN) if both β_k and β_p, are LN. Evidently, this facilitates calculation of WTP values. Meijer and Rouwendal [2006] show that if $\beta_i \sim exp(b_i)$ and $b_i \sim N(\mu_i, \sigma_i^2)$ for $i = k, p$, then

$$WTP_k = -\frac{\beta_k}{\beta_p} = -\frac{exp(b_k)}{exp(b_p)} = -exp(b_k - b_p). \tag{5}$$

Hence, WTP_k is lognormally distributed because $b_k - b_p$ is normally distributed.[1] Therefore, one obtains $WTP_k \sim LN(\tilde{\mu}_k, \tilde{\sigma}_k^2)$ where $\tilde{\mu}_k = \mu_k - \mu_p$ and $\tilde{\sigma}_k^2 = \sigma_k^2 + \sigma_p^2$.[2] Hence, one can use the means (μ_k, μ_p) and standard deviations (σ_k, σ_p) estimated from the random-coefficient model to derive the WTP distribution.

Unfortunately, the ratio of two mixing distributions of the N type does not result in parametric distribution. Therefore, the coefficient of the price attribute, β_p, is usually treated as nonstochastic to be able to divide the numerator N mixing distribution by a scalar (see Hole [2008]). Parametric calculation of the WTP values of random-coefficient models is not possible as long as not both coefficients – β_k and β_p – are LN-distributed. One solution to this problem is to simulate the WTP values using the estimated mixing distributions.[3] Most common software packages feature random numbers drawn from standard distributions such as N and LN. Thus, first one can draw random numbers from the N or LN mixing distribution (specified by the estimated parameters of the random-coefficient model). Using these draws the WTP values are then again given by dividing each single draw for β_k and β_p by each other according to Eq. (4).

Some Illustrative Examples

We start with an example where both the coefficient of the k'th attribute and the price attribute are LN. This permits a comparison between the parametrically derived and simulated WTP values. The estimated parameters of LN-mixing distributions $\hat{\mu}_k$ and $\hat{\sigma}_k$ are usually the mean and the standard deviation of the underlaying normal distribution. Hence, we have to calculate the mean, median and standard deviation of the WTP values themself. According to Train [2003] and Hole [2008], the statistics of the WTP values are given by

$$\begin{aligned} WTP_{\text{mean}} &= exp(\tilde{\mu}_k + \frac{\tilde{\sigma}_k^2}{2}) \\ WTP_{\text{median}} &= exp(\tilde{\mu}_k) \\ WTP_{\text{sd}} &= exp(\tilde{\mu}_k + \frac{\tilde{\sigma}_k^2}{2})\sqrt{exp(\tilde{\sigma}_k^2) - 1}. \end{aligned} \tag{6}$$

where $\tilde{\mu}_k$ and $\tilde{\sigma}_k$ are the mean and the standard deviation of the LN distributed WTP values as seen above.

[1] The so-called convolution of two independent normally distributed random variables is a normal distribution. That means that the difference of two normally distributed random variables is normally distributed.

[2] The coefficients are assumed to be independent of each other and therefore, the covariance is zero and drops out in the calculation.

[3] Another way to obtain the WTP values in this case is to estimate individual-specific coefficients conditional on the individuals actual choice (see Train [2003] and Greene et al. [2005]).

Alternatively, one can simulate these statistics by sampling a sufficient high number of random draws from the mixing distributions of the coefficients β_k and β_p. In this example, we have

$$\beta_k \sim LN(\mu_k, \sigma_k^2), \quad \beta_p \sim LN(\mu_p, \sigma_p^2), \tag{7}$$

where μ_k, μ_p, σ_k^2 and σ_p^2 are the estimated parameters from the random-coefficient model. According to Drukker and Gates [2006], the more uniform the coverage over the domain of integration, the better the numerical approximation. Therefore, we compare random draws from the mixing distribution with so-called Halton draws (see Train [2003], Ch.9). Halton draws divide the zero to one range into segments of equal size. We use these draws to calculate the quantiles from the mixing distributions and simulate the WTP values from these draws.

The WTP values are calculated according to Eq. (4) for each single draw as the draws are independent from each other. In the case of Halton draws, they first have to be randomized as they are ordered by construction. The descriptive statistics of these individual values approximate the parametric values as the number of draws increases. Here, we set $\mu_k = -0.8$, $\mu_p = -3.5$, $\sigma_k = 1.1$ and $\sigma_p = 0.8$.[4] Table 1 compares the parametrically and the simulated WTP values. It shows that the simulated values approximate the parametric values with 10^4 random draws and practically converge with 10^6 draws. The Halton draws perform slightly better, especially in the case of the standard deviation. The second example illustrates the simulation of WTP values if $\beta_k \sim N$ and

Statistic	Parametric	Random Draws		Halton Draws	
		10^4 Draws	10^6 Draws	10^4 Draws	10^6 Draws
WTP_k^{mean}	37.525	37.419	37.513	37.413	37.526
		(-0.28%)	(-0.03%)	(-0.30%)	(0.00%)
WTP_k^{median}	14.880	14.856	14.880	14.887	14.881
		(-0.16%)	(0.00%)	(0.05%)	(0.01%)
WTP_k^{sd}	86.875	84.071	86.538	84.092	86.895
		(-3.23%)	(-0.39%)	(-3.20%)	(0.02%)

The difference between parametric and simulated values (in %) are given in parentheses.

Table 1: Parametric and simulated WTP_k values

$\beta_p \sim LN$, making a parametric derivation impossible. One can again draw random numbers from the specified N and LN distributions to obtain the statistics of the WTP values for attribute k. This time, $\mu_k = -1.4$ and $\sigma_k = 1.9$. The distribution of the WTP values is depicted in Figure 1, with mean -63.85 (-63.83) CHF, median -37.5 CHF, and standard deviation 133.81 (133.71) CHF. The values are obtained from 10^6 Halton draws and 10^6 random draws (in parentheses). It is visible to the naked eye that the functional form of the resulting WTP_k distribution is neither N nor LN.

[4] These values are taken from an ongoing discrete choice analysis conducted at the Socioeconomic Institute of the University of Zurich.

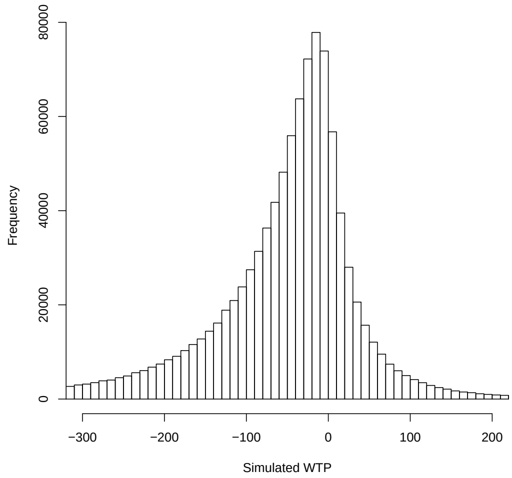

Figure 1: Histogram of simulated WTP values

Conclusion

Random-coefficient models used in Discrete Choice Experiments (DCEs) permit the estimation of entire distributions of willingness-to-pay (WTP) values. However, WTP values amount to a ratio of two estimated mixing distributions, which presents problems unless both are lognormally distributed. Simulation using both random and Halton draws provides a solution to this. In both examples presented, especially the Halton draws perform very well. Thus, it is possible to obtain accurate WTP values for a normally distributed attribute while the price attribute is assumed to be lognormal by simulation.

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